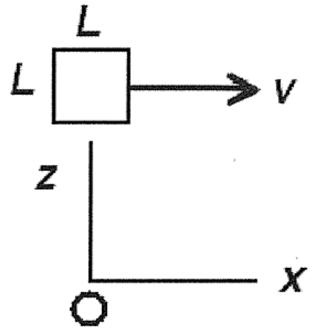
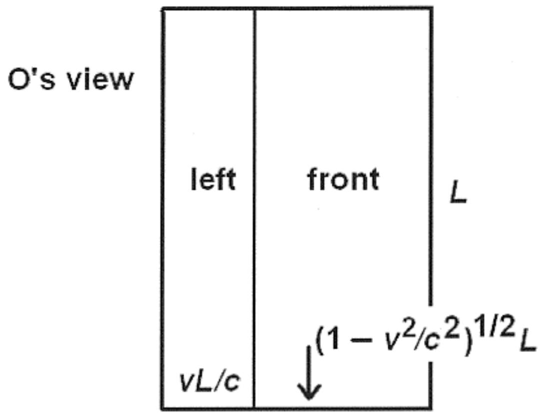
Visual Appearance of a Passing Cube



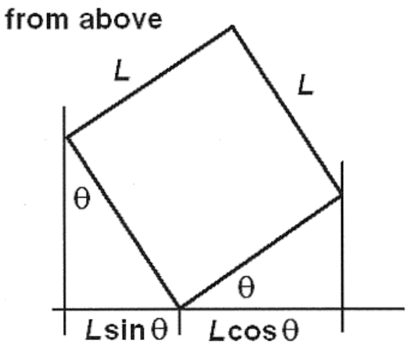
An observer O is stationary at the origin of an inertial reference frame (x, y, z). A cube of side L, oriented with edges parallel to the axes moves in the x-z plane with constant speed v parallel to (but not along) the x axis. We'll assume that the cube is sufficiently small and/or far away that all light rays from it to O can be considered parallel. But light travel time across the cube will be allowed for.

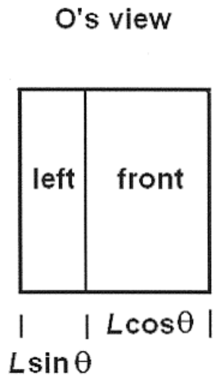
Consider the instant when O sees the closest of the cube's sides as being face on. Light from the rear of the cube has travelled an extra distance L to reach the observer at this instant. It must have been emitted a time L/c earlier than light from the front face, when the cube was further to the left by distance vL/c, all rays being considered parallel.



So O does not observe the cube as it was at some instant in its own time. Rather, O observes photons emitted over a range L/c of time. Therefore O instantaneously sees the left face of the cube spread out to the left of the front face, and occupying a width vL/c.

But photons recorded instantaneously by O from the cube's front face did leave that face simultaneously in O's time (assuming parallel rays). There is no differential time delay, so the requirement for relativistic contraction is met: O sees that face as having length $(1 - v^2/c^2)^{1/2}L$ in its direction of travel.





Imagine that we could neglect both light travel time across the cube and Lorentz-FitzGerald contraction and suppose we rotate the cube through an angle θ about the z axis. Then O would see the projected front and left faces as shown. This would match the view calculated above provided we choose:

$$\theta = \arcsin(v/c) = \arccos\left[(1 - v^2/c^2)^{1/2}\right] .$$

Because of light travel time variation across the object, O sees it as rotated through this angle.

We note the following **key points**:

- Lorentz contraction is based on an observer making simultaneous position fixes of two locations on the moving object; visual appearance is based on simultaneous photon arrivals at the detector.

- Light travel time across the object – a pre-relativistic or semi-relativistic effect measured by v/c – produces an apparent rotation of a non-infinitesimal volume.

- Lorentz-FitzGerald contraction – a relativistic effect measured by v^2/c^2 – is present, but is embedded in the apparent rotation of pre-relativistic origin.

- There is, for the configuration examined, no contraction of the volume – the cube appears rotated, not contracted.

Historically, it seems that physicists were so taken by Lorentz-FitzGerald contraction that nobody thought about the effects of light travel time and aberration on the visual appearance of moving bodies until the 1950s.

Original references

McCrea, W.H. (1952), The FitzGerald-Lorentz contraction, some paradoxes and their resolution, *The Scientific proceedings of the Royal Dublin Society*, Vol. 26 (n.s.), No. 1, pp. 27–36.

Penrose, R. (1959), The apparent shape of a relativistically moving sphere, *Proc. Camb. Phil. Soc*., vol 55(1), pp 137–9.

Terrell, J. (1959), Invisibility of the Lorentz contraction, *Phys. Rev*., vol 116, pp 1041–5.

R. Burman, 02 May 2013

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