Physics Honours & Masters 2013

University of Western Australia, Australian National University

Relativistic Electrodynamics – Assignment 5

The assignments are of equal value.

Deadline for UWA students: Friday 31 May, 5 pm, in labelled box on the First Floor.

Attempt all questions. The two problems are of equal value.

Any combination of computer and traditional mathematical methods may be used.

A11. Energy transfer in relativistic collisions. An electron, of rest mass $m_{\rm e}$ and charge $-e$, approaches a positive ion of rest mass M and charge Ze from a large distance. In the initial rest frame of the ion, the **impact parameter** (minimum separation of the ion from the asymptote to the incoming trajectory of the electron) is b. The electron has initial speed v and initial Lorentz factor $\gamma(v)$ in this frame. The **scattering angle** – the angle Θ through which the electron is deflected – is defined as the angle between the asymptotes to the outgoing and incoming legs of the electron's trajectory in this frame. Classical (non-quantum-mechanical) analysis shows that Θ is uniquely determined by b according to

$$\tan\left(\frac{\Theta}{2}\right) = \frac{1}{\gamma}\,\frac{Ze^2}{m_{\rm e}v^2}\,\frac{1}{b}\,.$$

(i) Draw a diagram to illustrate this electron-ion scattering event, clearly labelling the various quantities described above.

(ii) For elastic collisions, it can be shown that the 4-momentum transfer squared (a Lorentz invariant) is given by (see Section 19 on *Relativistic Collisions* in our RED-MER notes):

$$Q^2 = 4(\gamma m_{\rm e} v)^2 \sin^2(\Theta/2) \qquad \text{(elastic collisions)}\,.$$

Use the $\Theta(b)$ relation above to express Q as function of b for elastic collisions.

(iii) Show that the energy transfer $T(b)$, assuming the collision to be elastic, is given by

$$T(b) = \frac{2Z^2e^4}{m_{\rm e}v^2}\,\frac{1}{b^2 + {b_{\rm m}}^2}\,,$$

where $b_{\rm m} = (Ze^2)/(\gamma m_{\rm e} v^2)$. Hint: you can use a result for the energy transfer in elastic collisions from Jackson's problem 11.26 (pp 576–7). Express $T(0)$ in terms of γ (not v.)

Note: These calculations provide a basis for improvement to the simplified treatment of bremsstrahlung in lectures, which used a straight-line trajectory for large impact parameter.

A12. Energy transfer to an electron. An ion (rest mass M and charge Ze) travels past an electron (rest mass m_{e} and charge $-e$) on an almost straight-line trajectory, of large impact parameter b, at relativistic speed v.

(i) Calculate the small transverse impulse given to the nearly stationary electron by the electric field of the passing relativistic ion.

(ii) Calculate the energy transfer $T \approx (\Delta p)^2/(2m_{\mathrm{e}})$ to the electron as a function of b. Compare your result with the less restrictive result obtained in problem **A11**.

RB 20 May 2013

NB: The next assignment, Assignment 6, will be the final assignment.