Physics Honours & Masters 2013

University of Western Australia, Australian National University

Relativistic Electrodynamics – Assignment 4

The assignments are of equal value.

Deadline for UWA students: Wednesday 22 May, 5 pm, in labelled box on the First Floor.

Attempt all questions. The three questions are worth 10 marks each.

Any combination of computer and traditional mathematical methods may be used.

A8. Magnetic stress tensor. A stress 3-tensor in some region of space has components

$$t_{mn} = B_m B_n - (1/2)B^2\delta_{mn},$$

where $(B_k) \equiv \mathbf{B}$ is a 3-vector field of magnitude B. (This tensor is, within a constant factor, the magnetic stress 3-tensor used in, for example, magnetohydrodynamics.)

(i) Verify that, at any point, $\mathbf{B}$ is an eigenvector of (t_{mn}). What are the eigenvalues of (t_{mn})?

(ii) Show that the stress at any point can be represented as a tension $B^2/2$ along $\mathbf{B}$ together with a two-dimensional pressure $B^2/2$ perpendicular to $\mathbf{B}$. Illustrate with a sketch.

(iii) Show that the stress at any point can alternatively be represented as a tension B^2 along $\mathbf{B}$ together with an isotropic (three-dimensional) pressure $B^2/2$. Illustrate with a sketch.

A9. Orthogonal E and B. (i) Show that if $\mathbf{E}$ and $\mathbf{B}$ are orthogonal and $E < cB$ (SI units), then it is possible to find an inertial frame in which there is no electric field.

(ii) Show that if $\mathbf{E}$ and $\mathbf{B}$ are orthogonal and $cB < E$ (SI units), then it is possible to find an inertial frame in which there is no magnetic field. What about the case $E = cB$?

see over for Problem A10 . . .

A10. An exercise in integration. A time-varying electromagnetic field, of limited duration, acts on a particle of mass m and charge e that is harmonically bound with natural frequency ω_0 and a small damping constant (inverse time constant) Γ. The bound charge responds with non-relativistic motion that is of small amplitude on the scale of the spatial variation of the disturbing field. We are given the following result, in terms of the electric field spectrum, for the energy transferred to the oscillator as calculated in the frequency domain:

$$\Delta E = \frac{e^2}{m}\left(\int_{-\infty}^{\infty} \frac{i\omega|\tilde{\mathbf{E}}(\omega)|^2}{{\omega_0}^2 + i\omega\Gamma - \omega^2}\, d\omega\right) = \frac{i\, e^2}{m}\left(\int_{-\infty}^{\infty} \frac{s\,|\tilde{\mathbf{E}}(\omega(s))|^2}{1 - s^2}\, ds\right) .$$

The second form neglects damping and uses $s \equiv \omega/\omega_0$ as a dummy variable of integration. By giving s a small imaginary part, use contour integration in the complex s plane with Cauchy's residue theorem to evaluate this singular integral to obtain

$$\Delta E = \frac{\pi\, e^2}{m}|\tilde{\mathbf{E}}(\omega_0)|^2 \,,$$

for negligible damping.

Explain each step in your integration. Note any assumptions made about the function $|\tilde{\mathbf{E}}(\omega)|^2$ in obtaining this result. The *Wikipedia* article *Methods of Contour Integration* may be a useful reference.

RB 09 May 2012