Physics Honours & Graduate Course, 2013

University of Western Australia, Australian National University, Monash University

Relativistic Electrodynamics -- Assignment 3

The assignments are of equal value.

Deadline for UWA students: Friday 10 May, 5 pm, in labelled box on the First Floor.

Just ONE question this time. The 5 parts (a) to (e) are worth 6 marks each.

A7. Spectrum from a passing charge. Formulas on page RED37 give the field components of an un-accelerated passing charge as a function of time, expressed in dimensionless form.

(a) Perform a Fourier analysis of those results to express them in the frequency domain.

Hint: The results involve the modified Bessel functions $K_0(x)$ and $K_1(x)$, with arguments a dimensionless frequency, $x \equiv b\omega/(\gamma u)$. You may obtain the required Fourier integrals by using a computer program such as *Maxima* (a free download) or *Mathematica*, or by looking up tables, or from Jackson's *Classical Electrodynamics*.

(b) Use the asymptotic (large-x, $x \to \infty$) approximations for $K_0(x)$ and $K_1(x)$ (obtained from Jackson or elsewhere) to examine the high-frequency behaviour of the spectra of the three field components, explaining quantitatively what you mean by "high".

(c) Obtain (utilizing Jackson or otherwise) power series approximations for $K_0(x)$ and $K_1(x)$ valid for small x ($x << 1$); note the log term in $K_0(x)$. Hence examine the low-frequency behaviour of the spectra of the three field components, explaining quantitatively what you mean by "low". Check that you have kept sufficient terms in the $K_0(x)$ and $K_1(x)$ expansions to make the spectral behaviour clear.

(d) Using your results from (b) and (c), make sketches of what you expect the spectra to look like. Indicate on the sketches features corresponding to what you found in (b) and (c).

(e) Produce computer-generated graphs showing what the spectra look like. Comment on their features as related to your calculations in (b) and (c). [Marks for this part will depend on the visual quality of the figures, including labelling.]

For information on Bessel functions, see Jackson's Sect. 3.7, pp 111–116, *Laplace's Equation in Cylindrical Coordinates; Bessel Functions*.

RB, 24 Apr 2013