Physics Honours & Graduate Course, 2013

University of Western Australia, Australian National University, Monash University

Relativistic Electrodynamics -- Assignment 2

The assignments are of equal value.

Deadline for UWA students: Friday 03 May, 5 pm, in labelled box on the First Floor.

Attempt all questions. The three questions are worth 10 marks each.

Any combination of computer and traditional mathematical methods may be used.

A4. Uniformly moving charge – "flattening" of the E field. As a measure of the relativistic flattening of the electric field of a charge moving at constant velocity **u**, we introduce the angle α between two conical surfaces that include between them half of the total electric flux. That is, half the flux through a sphere in the observer's frame is contained in the "equatorial zone" between $\theta = (\pi + \alpha)/2$ and $\theta = (\pi - \alpha)/2$, **u** being the axis. Consider only the extreme relativistic case, $\gamma >> 1$, so only angles θ such that $\theta = \pi/2 - \varepsilon$, with $|\varepsilon| << 1$, need be considered.

Start with the result of Exercise 8, p RED40: that the equation for the electric field strength of a uniformly moving charge q, namely

$$E = (Kq/r^2)(1 - \beta^2)/(1 - \beta^2\sin^2\theta)^{3/2} ,$$

can be approximated for $\gamma^2 >> 1$ and $\varepsilon^2 << 1$, in the relevant angular range, by

$$E \approx (Kq/r^2)\gamma/(1 + \gamma^2\varepsilon^2)^{3/2} .$$

(a) Let ε range from $-\alpha/2$ to $+\alpha/2$ and integrate to obtain the flux through the narrow equatorial belt and deduce the value of α in terms of γ. Illustrate the geometry of the belt region with a sketch.

(b) If the charge passes a stationary observer at impact parameter b, calculate the peak E observed and estimate the duration T of the observed pulse. If a 7-GeV electron (rest mass 0.5 MeV/c^2) passes 1 m away from an observer, estimate, to order of magnitude only, T and the peak E observed.

A5. Some field geometry – continuation of A2. Begin with the solution for $\beta(\alpha, \varepsilon)$.

(i) Simplify $\beta(\alpha, \varepsilon)$ for the case of equal electric and magnetic field strengths, $\varepsilon = 1$.

(ii) Develop a simple approximate solution applicable for the magnetically dominated case, $\varepsilon << 1$.

(iii) Develop a simple approximate solution applicable for the electrically dominated case, $\varepsilon >> 1$.

(iv) Plot curves of β against ε, for several choices of α.

See over for Problem A6 . . .

A6. Galilean transform of wave equation. Consider two inertial reference frames S and S′, having spatial coordinates (x, y, z) and (x', y', z') respectively, with S′ moving at velocity **v** relative to S′ along their common x-x' axes; their origins coincide at time $t = 0 = t'$. A scalar function of position and time can be expressed in terms of either set of coordinates:

$$\Psi(\mathbf{r}, t) = \Psi'(\mathbf{r}', t') , \qquad \text{with } \mathbf{r} = (x, y, z) , \quad \mathbf{r}' = (x', y', z') ,$$

meaning that the two mathematical functions, Ψ and Ψ', representing some scalar physical quantity, have the same value at any given space-time point. Suppose that S and S′ are connected by a galilean transformation:

$$x', \; y', \; z', \; t' \; = \; x - vt, \; y, \; z, \; t , \qquad v \equiv |\mathbf{v}| .$$

(a) Use standard multivariable calculus to determine how the components of the gradient operator, ∇, and the time derivative $\partial/\partial t$ transform between these frames.

(b) Determine how the wave operator or d'Alembertian, $\Box^2 \equiv c^{-2}\partial^2/\partial t^2 - \nabla^2$, transforms between S and S′.

(c) Show that the wave equation $\Box^2\Psi = 0$ takes the following form in S′:

$$[\, \Box'^2 + c^{-2}(\mathbf{v}\bullet\nabla')^2 - 2c^{-2}(\mathbf{v}\bullet\nabla')\partial/\partial t' \,]\Psi' \; = \; 0 .$$

(d) Use this result to obtain a formula for the non-relativistic Doppler effect.

RB, 18 April 2013