Physics Honours & Masters 2013

University of Western Australia, Australian National University

Relativistic Electrodynamics – Assignment 6

The assignments are of equal value.

Deadline for UWA students: Monday 10 June, 5 pm, in labelled box on the First Floor.

Attempt all questions. The two problems are of equal value.

Any combination of computer and traditional mathematical methods may be used.

A13. Energy transfer to a bound charge. A time-varying electromagnetic field, of limited duration, acts on a particle of mass m and charge $-e$ that is harmonically bound with natural angular frequency ω_0 and a small damping constant (inverse time constant) Γ. The bound charge responds with non-relativistic motion of small amplitude on the scale of the spatial variation of the disturbing field.

(a) Write down the equation of motion for a damped forced harmonic oscillator and take a Fourier time transform. Hence, referring to results from the previous assignment, obtain the following formulas, in terms of the electric field spectrum, for the energy transferred to the oscillator as calculated in the frequency domain:

$$\Delta E = \frac{e^2}{m}\left(\int_{-\infty}^{\infty} \frac{i\omega|\tilde{\mathbf{E}}(\omega)|^2}{{\omega_0}^2 + i\omega\Gamma - \omega^2}\, d\omega\right) = \frac{i\, e^2}{m}\left(\int_{-\infty}^{\infty} \frac{s\,|\tilde{\mathbf{E}}(\omega(s))|^2}{1 - s^2}\, ds\right),$$

where the second form neglects damping and uses $s \equiv \omega/\omega_0$ as a dummy variable of integration. By using the integral evaluated in an earlier assignment (or otherwise), obtain the following result for the energy transferred to the mass m for negligible damping:

$$\Delta E = \frac{\pi\, e^2}{m}|\tilde{\mathbf{E}}(\omega_0)|^2 .$$

(b) Suppose that the applied field is from a charge moving at constant relativistic velocity $\mathbf{u}$ and Lorentz factor $\gamma(u)$ passing with impact parameter b. Its electric field components at the oscillator have Fourier transforms:

$$E_\perp(\omega) = \frac{Ze}{bv}\left(\frac{2}{\pi}\right)^{1/2} \xi K_1(\xi)\,, \qquad E_\parallel(\omega) = -i\frac{Ze}{\gamma bv}\left(\frac{2}{\pi}\right)^{1/2} \xi K_0(\xi)\,,$$

where $\xi \equiv \omega b/(\gamma v)$. Obtain an expression for ΔE as a function of ξ. Examine its behaviour for large ξ, stating quantitatively what you mean by 'large'. Discuss the physical significance of this case.

A14. Spectrum from a passing charge (continuation of A7). Suppose that the moving charge and the observation point are embedded in an infinite dielectric of permittivity $\epsilon(\omega)$ such that $\beta^2\epsilon(\omega) < 1$. We are given that the frequency spectrum of the electric field component perpendicular to the motion is now:

$$E_\perp(\omega) = \frac{q}{u}\left(\frac{2}{\pi}\right)^{1/2} \frac{\lambda}{\epsilon(\omega)} K_1(\lambda b)\,, \qquad\qquad \lambda = |\omega|(1-\beta^2\epsilon)^{1/2}\rho/u\,.$$

Take the dielectric to be non-dispersive and apply an inverse Fourier transform to obtain the time-domain field component $E_\perp(t)$.

Note: This work particularly relates to **Cherenkov radiation**, emitted by a charged particle – even if moving at constant velocity – travelling through a medium at a speed greater than the phase velocity of the radiation in that medium.

RB 28 May 2013